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# Advances in Inverse Groundwater Modeling: A Comprehensive Review

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## ABSTRACT

The management and sustainable use of groundwater resources are critical components in addressing global water challenges. In this context, inverse groundwater modeling has emerged as a powerful tool for characterizing subsurface properties, optimizing resource utilization, and mitigating the impacts of anthropogenic activities on aquifers. This review paper provides a comprehensive and up-to-date survey of the advancements in inverse groundwater modeling techniques, methodologies, and applications. The paper begins by presenting an overview of the fundamental principles underlying inverse modeling, elucidating the mathematical frameworks and numerical algorithms employed in estimating subsurface parameters. It explores various geophysical and hydrogeological data types commonly utilized in inverse modeling, such as hydraulic head measurements, and geophysical surveys. The integration of multiple data sources for enhancing model reliability and reducing uncertainty is also discussed. Furthermore, the review highlights recent developments in regularization techniques, sensitivity analysis, and uncertainty quantification within the context of inverse groundwater modeling. Case studies from diverse hydrogeological settings illustrate the practical applications of these methodologies in real-world scenarios, showcasing their efficacy in addressing complex groundwater management challenges, including contaminant transport, aquifer recharge, and sustainable resource exploitation. The review concludes by outlining current research gaps and future directions in the field of inverse groundwater modeling, emphasizing the need for interdisciplinary collaboration, data integration, and advanced computational approaches. This synthesis of contemporary knowledge serves as a valuable resource for researchers, practitioners, and policymakers engaged in groundwater management and environmental sustainability.

## Introduction

According to the World Bank (2009) water table of 60% of the aquifer will be endangered within 15-20 years in India due to overexploitation of groundwater for agricultural purposes. The groundwater simulation model is a potent possible technique among other ways to investigate and understand the groundwater dynamics, which effectively simulates the response to stress in the system. It can be applied as a management tool for adopting different policies to seek the best among them and works as a future scenario predictor (Zhou and Li, 2011). The reliability of groundwater state (groundwater head) estimation depends upon the accuracy of estimated sub-surface parameters. These aquifer parameters are transmissivity, hydraulic conductivity, storage coefficient and dispersivity (Rastogi, 2012). The pumping test and graphical matching are the oldest and most common techniques to determine these parameters. However, these techniques are based on governing equations with closed form solution and are generally restricted to homogeneous and isotropic aquifer domains (Theis, 1935).

In the recent years, an advanced, economically-feasible and automatic technique called inverse groundwater modelling has become a widely used mathematical practice to estimate aquifer parameters (Figure 1).

Using inverse problem, distributed parameters are assigned to a mathematical model with known boundary conditions in such a way, that it minimizes the error between the observed and simulated state variables (Lakshmi Prasad and Rastogi, 2001). Many researchers widely applied the inverse problem to explore different areas in groundwater like, parameter estimation (Nelson 1960, 1961; Yoon and Yeh 1976; Cooley, 1982; Hoeksema and Kitanidis, 1985; Mahinthakumar and Sayeed, 2005; Yaoand Guo, 2014; Yeh and Yoon, 1981), contaminant source identification (Aral *et al.*, 2001; Mahar and Datta, 2001, 1997; Snodgrass and Kitanidis, 1997) and coupled inverse problem (Sun and Yeh, 1990). Evolution of inverse problem with passage of time is presented in a detailed manner by Yeh (1986); Carrera *et al.,* (2005); Vrugt *et al.,* (2008); Zhou *et al.,* (2014) and Yeh (2015) in their review papers. The main scope of the present study is development of effective simulation-optimization (SO) model estimating aquifer parameters. The simulation models based on FEM and meshfree (Mfree) are developed and coupled with different metaheuristic based optimizations models and eventually are applied to a large aquifer system and the model accuracy and efficiency are compared.

The solution of inverse problem was more prominently investigated after 1960 with its application on geo-physical exploration. Neuman (1973) was the first person who classified these approaches on the basis of their solution strategy, as direct and indirect methods. Similarly, Chavent (1979) classified these two methods based on the perspective of error criteria and named them as equation error criteria method and output error criteria method respectively. The main objective of the direct method is to minimize this equation error using two approaches, which are matrix method and mathematical programming methods (Sun, 1999).

The matrix method acquires solution by inversion of a matrix, however, mostly the obtained solution smears due to ill- conditioning of matrix equations (Figure 2). Emsellem and Marsily (1971); Frind and Pinder (1973); Sagar *et al.,* (1975) and Yeh *et al.,* (1983) used matrix inversion approach for groundwater parameter estimation. In the mathematical programming approach, the inverse problem is converted into linear programming (LP) problem and aquifer state governing equations are utilized as constraints. This approach is based on the assumption that aquifer parameters are a linear function of error; therefore, the simplex method is employed to get optimum parameter values. Neuman (1973) and Hefez *et al.,* (1975) were some researchers that applied the linear programming for the inverse groundwater problem.

The indirect method can be classified based on solution technique used i.e. gradient based optimization and non- gradient based optimization approach. In the first approach, gradient improves the initial guess of parameter until the optimum value is achieved. Some examples of gradient based approaches are Levenberg- Marquardt method (Keidser and Rosbjerg, 1991), Gauss- Newton method (Kitanidis and Lane, 1985) and Conjugate gradient method (Carrera and Neuman, 1986). Generally, local optima based solution strategies don’t have the potential to tackle these kinds of peculiarities and have a larger probability to get stuck into the local minima.

In last two decades, a new paradigm shift is being observed in the field of groundwater management, due to the introduction of population-based stochastic search methods, which belongs to non-gradient based classification.

The advantages of these global search methods are: (1) These are based on the random search, hence it is able to explore the whole solution space (2) It doesn't require the objective function to be continuous, hence it can work efficiently on discrete problems (3) These are highly robust method with guaranteed global solution (Chiu, 2014).

Various methods of this class have been successfully applied in the field of groundwater, which are: genetic algorithm (GA) (Harrouni *et al.,* 1996; Lakshmi Prasad and Rastogi, 2001), simulated annealing (SA) (Zheng and Wang, 1996), particle swarm optimization (PSO) (Ch and Mathur, 2012), ant colony optimization (ACO) (Abbaspour *et al.,* 2001), artificial bee colony optimization (ABCO) (Li *et al.,* 2006), co- variance matrix evolution adaptation strategy (CMA-ES) (Elshall *et al.,* 2015), differential evolution (DE) (Chiu, 2014; Rastogi *et al.,* 2014) and cat swarm optimization (CSO) (Thomas *et al.,* 2018) amongst others.

## Literature review on inverse groundwater modelling

### Direct method

Nelson (1961, 1960) presented maiden work on the direct method for parameter estimation. Finite difference approximation is utilized for the inversion of flow equation along the streamline to calculate the hydraulic conductivity and transmissivity values. The projected method was applied to two and three-dimensional isotropic homogeneous confined and unconfined aquifers in the steady state.

Emsellem and De Marsily (1971) proposed an automatic direct method to calculate the transmissivity, storativity and recharge value using finite difference. He introduced the regional distribution concept of aquifer parameters. The main aim of this study was to minimize the norm of error of flow to estimate the mean value of different aquifer zones in two-dimensional steady and transient states.

Sagar *et al.,* (1975) proposed the direct method to identify the aquifer parameters in the anisotropic and non-homogenous aquifer, for the transient conditions.

The head data over unknown nodes were obtained using Lagrange and spline interpolation techniques. The governing groundwater flow equations were solved using algebraic approach and without iterative improvement.

Yeh *et al.,* (1983) presented a model to estimate the transmissivity values in the two-dimensional confined aquifer for the transient condition. They used more advanced technique like krigging for head values interpolation. Finite element method was employed to discretize the governing equation. This method was suitable for domain having a small dimension or less number of the parameters.

Irsa and Zhang (2012) presented a novel method to estimate aquifer parameters for steady state condition with unknown boundary conditions. This method is based on the potential theory technique which was successfully applied to general inverse problems. They also showed the accuracy of proposed method which increases with higher number of observation head data, lower observation error and grid refinement.

Jiao and Zhang (2016) used the technique introduced by Irsa and Zhang (2012) for the parameters estimation in contaminant transport problem.

As discussed by most of the authors, the direct method has limitation like requirement of large number of observation well data which is difficult to follow for large scale regional groundwater flow problems.

## Indirect method

Indirect methods utilizes the SO model to solve the inverse problem. In this method, the simulation and optimization models are solved simultaneously. Optimization model provides the input in terms of parameters to simulation model for calculation of simulated head values which eventually utilized to evaluate the objective function iteratively. SO model utilizes the gradient based or non-gradient based optimization approaches that are discussed in terms of literature review in the upcoming sections.

## Simulation technique used in groundwater

Wu *et al.,* (2010) developed a regional groundwater flow model coupled with a deformation model to simulate land subsidence, called modified merchant model. It was solved using Multidimensional iterative finite element method for land subsidence in Shanghai city due to overexploitation of water. This model was initially calibrated using 28,184 hydraulic head data and 26,732 deformation data of past 45 years (1961 to 2005). This calibrated model further used for prediction of the future scenario of land subsidence up to the year 2020.

Paris *et al.,* (2010) modeled an aquifer system in Venice lagoon region using a complex three-dimensional finite element model (FEM) using FE-FLOW 5.3. This model analysis was carried out (1) to study the effect of the cutoff wall on the hydrologic regime; (2) to mitigate the related inundation hazard to prevent the erosion of polluted land and discharge of contaminant groundwater with surface water in the lagoon area. Whole aquifer domain was discretized by 3-D mesh consisting of 310,000 nodes and 550,000 triangular prismatic elements. This model was initially applied on a regional regime and then on the local regime to study the drained groundwater volume along a 5-km long bank of a harbor canal. Simulation results indicated, the decline in sub surface discharge in lagoon area is up to 85% and 1-m increment in the water table in the inland city of Mestre, Italy due to the construction of the cut-off wall.

Mondal *et al.,* (2010) developed a SO model for the treatment of an Aquifer located at Vadodara, Gujarat, India affected by total dissolved solid (TDS) as the main pollutant for identifying the best pump and treat policy. Simulation of groundwater and contaminant transport was performed by Finite element approximation, which was further coupled with non- dominant sorting genetic algorithm-II (NSGA-II) for multi-objective optimization.

The model was developed for minimizing cost function and time period for the remediation of the aquifer subject to bounds on pumping rates, groundwater heads, and concentration levels of the contaminant at all nodes of aquifer domain. Further three scenarios of a combination of pumping through abstraction wells and flushing through four recharge ponds were considered with different locations of extraction wells.

Hendricks Franssen *et al.,* (2011) presented first sub surface flow model, which is able to assimilate the real-time daily data of piezometric head for aquifer characterization. The obtained results were better than inversely calibrated data, which uses historical data and doesn't get it updated. In the proposed approach the aquifer parameters i.e. hydraulic conductivity and leakage coefficients are updated using Enhanced Kalman filter (EnKF) to get the head values for the Water Works Zurich and the simulation period of January 2004 to December 2007. The whole three-dimensional simulation model is discretized using Galerkin finite element method.

Engeler *et al.,* (2011) developed a 3-D finite element model of coupled groundwater flow and heat transport for Hardhof in Limmat Valley, Zurich (Switzerland). Hydraulic conductivity of aquifer and leakage coefficient of river bed both were estimated using pilot point inverse modeling. The temperature was used as a common linking variable between hydraulic conductivity and storage coefficient for coupling. The obtained results (temperature and head) showed greater agreement in three pumping wells and seven piezometric locations.

Banerjee *et al.,* (2011) applied an artificial neural network (ANN) for safe pumping rate assessment to uphold salinity in Kavaratti Island aquifer, India. For the forecasting of salinity for different pumping rate, feed forward ANN model with quick propagation (QP) as training algorithm with 2 years of real- time field data had been used. Prediction on water quality with varying pumping rate was made for a span of 5 years. The output obtained from ANN model was further compared with real field data and predicted data through SUTRA (FEM). Due to the nonlinear nature of ANN, it showed exceptional convergence for analyzing real- world data.

Radu *et al.,* (2011) performed a comparative study using different numerical schemes i.e. Galerkin finite element (GFE), Mixed hybrid finite element (MHFE) and finite volume (FV) for simulation of contaminant transport for heterogeneous porous media and quantified the numerical diffusion for the different schemes and its dependency on the Peclet number. The study showed that for the real problems (i.e. heterogeneity in parameter distribution), the differences between lower and higher order schemes were negligible but at same time computational cost increased.

Chandio *et al.,* (2013) studied the different remedial measures to prevent waterlogging condition due to seepage through Rohri canal on 1,000 ha of agricultural land near the Gambat railway station, district Khairpur, Sindh, Pakistan. To study the problem a 3-D finite element groundwater flow and solute transport based tool FEMGWST (developed at Universiti Putra Malasia) was used to evaluate the effectiveness of horizontal and vertical drainage systems, either independently or simultaneously. Ultimate results from analysis confirmed that combined drainage system is effective to reduce waterlogged area and it increased agricultural farmland for more crop yield.

Sherif *et al.,* (2014) developed 2-D finite element groundwater and solute transport model to study the salinity distribution in coastal aquifer of Wadi Ham, United Arab Emirates. The reason for seawater intrusion was a lack of rainfall and excessive pumping of groundwater which resulted in deterioration of groundwater quality, termination of domestic water supply and abandoned farms in nearby Fujairah city. The whole investigation was based on the transition zone between freshwater and seawater; and accordingly simulations were conducted in horizontal view under a transient condition with different pumping scenario.

## Optimization approaches used in indirect method

### Gradient-based optimization approach

Vemuri and Karplus (1969) presented an inverse model to estimate the hydraulic conductivity values in an unconfined aquifer. To minimize the objective function steepest descent algorithm was employed. Boundary conditions and storage coefficient values were fed to the inverse model and finite difference scheme was used to discretize the governing equation.

Cooley (1977) used non-linear least- square regression for an inverse problem for estimation of parameters like hydraulic conductivity (K), source-sink (Q) and flux values. Simulation of head values was performed by finite element method for confined aquifer domain in steady state condition. In this work non-linear system of the equation was solved by modified Gauss- Newton algorithm.

Chavent *et al.,* (1975) applied the optimal control theory to obtain the permeability distribution. To minimize non- quadratic criteria steepest descent method was used and for the same, the gradient was calculated using adjoint state method with available minimum computation time. The whole procedure was also followed for transmissivity estimation in two-dimensional aquifer domain.

Yeh and Yoon (1981) presented a synthetic model to estimate the optimum transmissivity values. Finite element method was used to discretize the two-dimensional governing equation. Later the values obtained through the forward model were fed to the modified Gauss- Newton optimization model to minimize the error based on least square criteria. The norm of covariance matrix was also calculated to test the reliability of the obtained parameters.

Sun and Yeh (1985) developed a model to structure identification and parameter estimation. The governing equation was discretized using finite element approximation. To obtain the nodal values of the gradient (sensitivity coefficient), the variational method was employed and later used for Gauss- Newton optimization method. They also used an automatic parameterization technique which was able to identify the zonation or continuous distribution of transmissivity.

Khan (1986) developed an inverse groundwater model consisting of unconstrained multivariate optimization (modified version of Newton`s second derivative method) and finite difference model for simulation of head values.

Li and Elsworth (1995) applied the Gauss-Newton method (GNM) for groundwater parameter estimation for transient condition. They used all three approaches i.e. influence coefficient method, sensitivity equation method and variational method to obtain the sensitivity coefficient. These all three methods were applied to a synthetic problem to get the better performance among them. It was found that GNM is performing well for a limited number of parameters.

Li and Yang (2000) proposed a new algorithm to minimize the objective function based on the generalized Gauss- Newton method for estimation of transmissivity. In this method, a scaling matrix was introduced to overcome the irregularity in weighting effect of residuals in each iteration.

By applying scaling matrix correction the results obtained were enthusiastic and performed well with less computational cost, provided scaling matrix is not being the identity matrix. This method encouraged to take a wide range of initial value of unknown parameters, with quick convergence. All these facts demonstrated the potential of the algorithm to solve inverse problems of more complicated nonlinear aquifer models naturally and quickly on the basis of finding suitable forms of the scaling matrix and the pre-conditioner.

Franssen *et al.*, (2003) presented a coupled inverse model of groundwater flow and mass transfer. Conjugant gradient and steepest descent algorithms were used to minimize the error function (sum of squared error in groundwater head and concentration value at each node). The sensitivity coefficients (derivative of objective function w.r.t. log conductivities, log storativities, prescribed heads at boundaries, retardation coefficients and mass sources) for Jacobin were calculated using adjoint state method. Presented model was successfully applied to a synthetic problem and it also improved the prediction of flow and aquifer characteristics if optimum amount of experimental data of groundwater head and concentration is available.

## Non- gradient based optimization approach

Aral *et al.*, (2001) presented the solution for contaminant source identification problem using the progressive genetic algorithm (PGA). The main objective of this problem was to minimize the residual between observed and simulated concentration values. The proposed method was found very efficient and cost effective as it reduces the repeated solutions.

Lakshmi Prasad and Rastogi (2001) used genetic algorithm (GA), as global optimization tool coupled with finite element method for simulation of groundwater head values in the intermediate points of confined aquifer problem. Initially coupled algorithm applied to a synthetic rectangular aquifer for estimation of transmissivity and the accuracy in results proved it to be a robust model. Later the same model was applied to a real field unconfined aquifer for estimation of hydraulic conductivity and recharge parameters. The solution obtained through GA model was compared with Gauss- Newton-Marquardt (GNM) method and proved superior.

**Table.1** Literature review for an inverse problem in groundwater using direct method.

| Governing equations | Numerical scheme | Parameter to be identified | Interpolation scheme | References |
|---|---|---|---|---|
| **Two and Three dimensional confined and unconfined aquifer (steady state)** | Finite difference method | Hydraulic conductivity and transmissivity | | (Nelson, 1960 and 1961) |
| **Two-dimensional confined aquifer (steady and transient state)** | Finite difference method | Transmissivity, Storativity, and Recharge | | (Emsellem and De Marsily, 1971) |
| **Two-dimensional anisotropic and non-homogeneous confined aquifer (Transient state)** | | Transmissivity, Storativity, and Recharge | Lagrange and spline interpolation | (Sagar et al., 1975) |
| **Two-dimensional unconfined aquifer (Transient )** | Finite difference method | Transmissivity | Krigging | (Yeh et al., 1983) |
| **Two-dimensional confined aquifer (steady)** | Based on potential theory technique | Hydraulic conductivity and Darcy’s flux | | (Irsa and Zhang, 2012) |
| **Two-dimensional unconfined aquifer (steady )** | Based on potential theory technique | Hydraulic conductivity, Darcy’s flux and concentration | | (Jiao and Zhang, 2016) |

**Table.2** Literature review for an inverse problem in groundwater using indirect method (Gradient-based approach).

| Governing equations | Numerical scheme | Parameter to be identified | Inverse- solution procedure | References |
|---|---|---|---|---|
| **2D confined aquifer (transient)** | Finite difference | Transmissivity, Storage coefficient and boundary of aquifer | Steepest descent method | (Vemuri and Karplus, 1969) |
| **2-D steady state** | Finite element | Hydraulic conductivity (k), Source-sink (Q) and flux | Modified Gauss- Newton (Non-linear regression) | (Cooley, 1977) |
| | Finite difference | Transmissivity and permeability | Steepest descent method | (Chavent et al., 1975) |
| **2D confined aquifer (transient)** | Finite element | Transmissivity | Gauss- Newton algorithm | (Yeh and Yoon, 1981) |
| **2D confined aquifer (transient)** | Finite element | Transmissivity | Gauss- Newton algorithm | (Sun and Yeh, 1985) |
| **2-D Unsteady state, Unconfined problem** | Finite difference | Hydraulic conductivity (k) | Powell`s algorithm<br>Fletcher- Powell`s algorithm<br>Newton-Khans algorithm | (Khan, 1986) |
| **2D confined aquifer (transient)** | Finite element | Transmissivity | Gauss- Newton algorithm | (Li and Elsworth, 1995) |
| **2-D Unsteady state, Unconfined problem** | Linear triangle finite element method | Transmissivity | Gauss- Newton method with improvement as incorporation of scaling matrix | (Li and Yang, 2000) |
| **Confined aquifer with solute transport equation** | Block center finite difference method | Transmissivity | Gradient-based methods | (Franssen et al., 2003) |

**Table.3** Literature review for inverse problem in groundwater using indirect method (Non- gradient based approach).

| Governing equations | Numerical scheme | Parameter to be identified | Inverse- solution procedure | References |
|---|---|---|---|---|
| **2-D unconfined aquifer flow and transport** | Finite element method | Source location and contaminant flow rate | Progressive genetic algorithm | (Aral et al., 2001) |
| **2-D Non-homogeneous confined and unconfined** | Finite element method | Transmissivity, Hydraulic conductivity and recharge parameter | Genetic Algorithm (GA) results were, compared with Gauss-Newton-Marquardt (GNM) method. | (Lakshmi Prasad and Rastogi, 2001) |
| **2-D homogeneous confined isotropic** | Finite element method | Pumping cost | Real-coded GA | (Yoon and Shoemaker, 2001) |
| **Contaminant transport through a saturated mono-dimensional porous medium processes** | | Velocity and dispersivity parameters | Genetic Algorithm | (Giacobbo et al., 2002) |
| **2-D homogeneous confined** | Finite difference (MODFLOW) | Transmissivity | GA, grid search and BFGS | (Tsai et al., 2003c) |
| **2-D Confined aquifer** | Galerkin finite element method | Transmissivity | Simulated Annealing (SA) | (Snehalatha et al., 2006) |
| **two- dimensional, heterogeneous-isotropic, confined aquifer** | Finite difference | Transmissivity | Genetic algorithm | (Ayvaz et al., 2007) |
| **Dore River basin (Unconfined aquifer)** | Analytical element method (AEM) | Groundwater management problem for minimization of discharge and pumping cost | Particle swarm optimization (PSO) | (Gaur et al., 2011) |
| **Three- dimensional, heterogeneous-isotropic, unconfined** | Finite difference (MODFLOW and MT3DMS) | Source identification | Simulated annealing | (Jha and Datta, 2012) |
| **Three- dimensional, heterogeneous-isotropic, unconfined** | Finite difference (MODFLOW and MT3DMS) | Source identification | Adaptive simulated annealing (ASA) | (Jha and Datta, 2013) |

| | | | | |
|---|---|---|---|---|
| **2-D Confined aquifer with contaminant transport equation** | FEM | Transmissivity, longitudinal dispersivity, transverse dispersivity. | Genetic algorithm (GA), Simulated annealing (SA) and Gauss-Newton-Marquardt (GNM) | (Rastogi and Huggi, 2013) |
| **2D Synthetic problem and real field problem (Pingtung Plain, Taiwan)** | FDM | Transmissivity and structural identification | Differential evolution (DE) | Chiu, (2014) |
| **Two- dimensional, heterogeneous-isotropic, confined (steady)** | Finite difference (MODFLOW and MT3DMS | Transmissivity | Differential evolution (DE) | (Gurarslan and Karahan, 2015) |
| **Three- dimensional, heterogeneous-anisotropic, unconfined** | Finite difference (MODFLOW and MT3DMS | sequential- monitoring network design and a source identification method | Simulated annealing | (Prakash and Datta, 2015) |
| **Two- dimensional, heterogeneous-isotropic, confined (steady)** | Finite difference (MODFLOW and MT3DMS | Source identification | Hybrid of binary GA and generalized reduced gradient method | (Ayvaz, 2016) |

**Fig.1** Inverse groundwater model

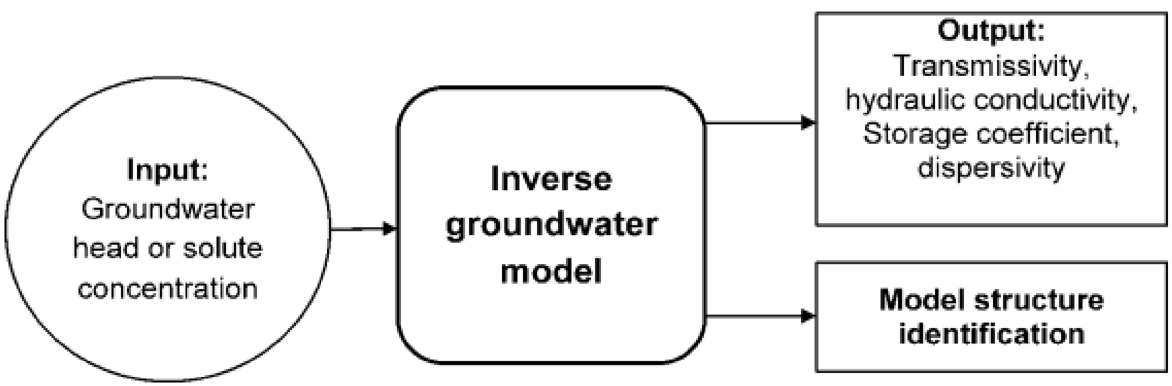


**Fig.2** Flowchart of the direct method based on matrix inversion approach.

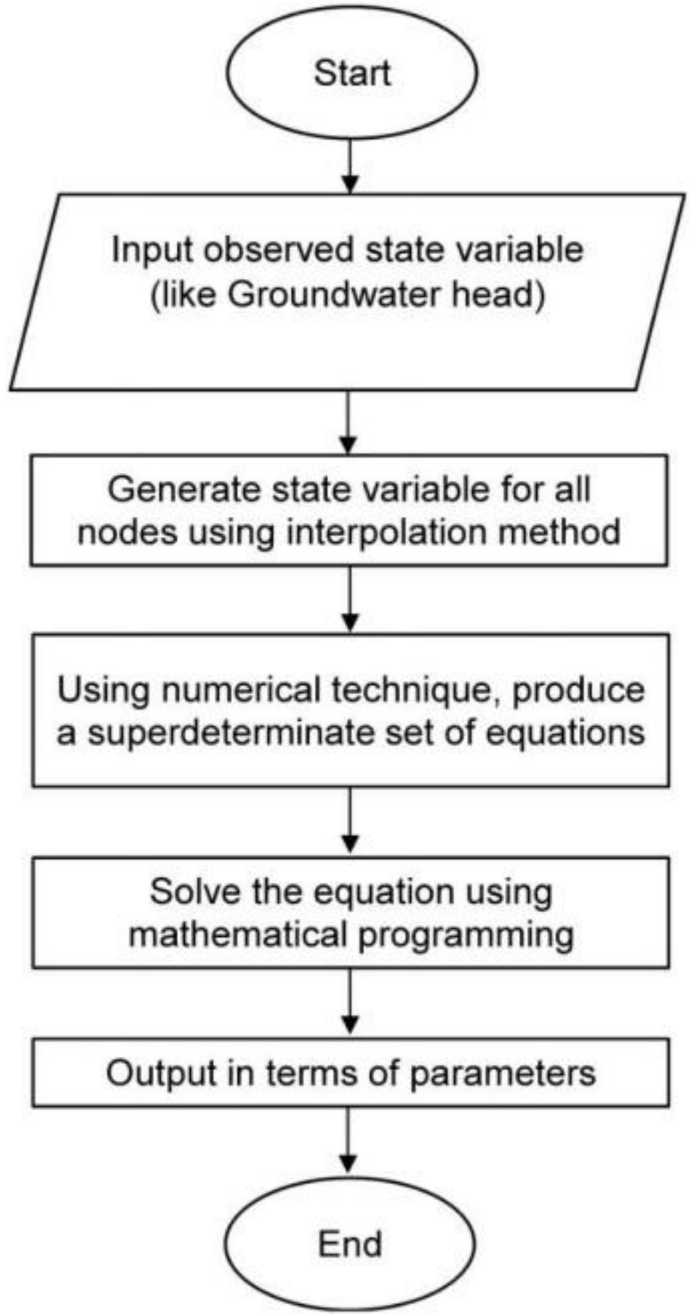
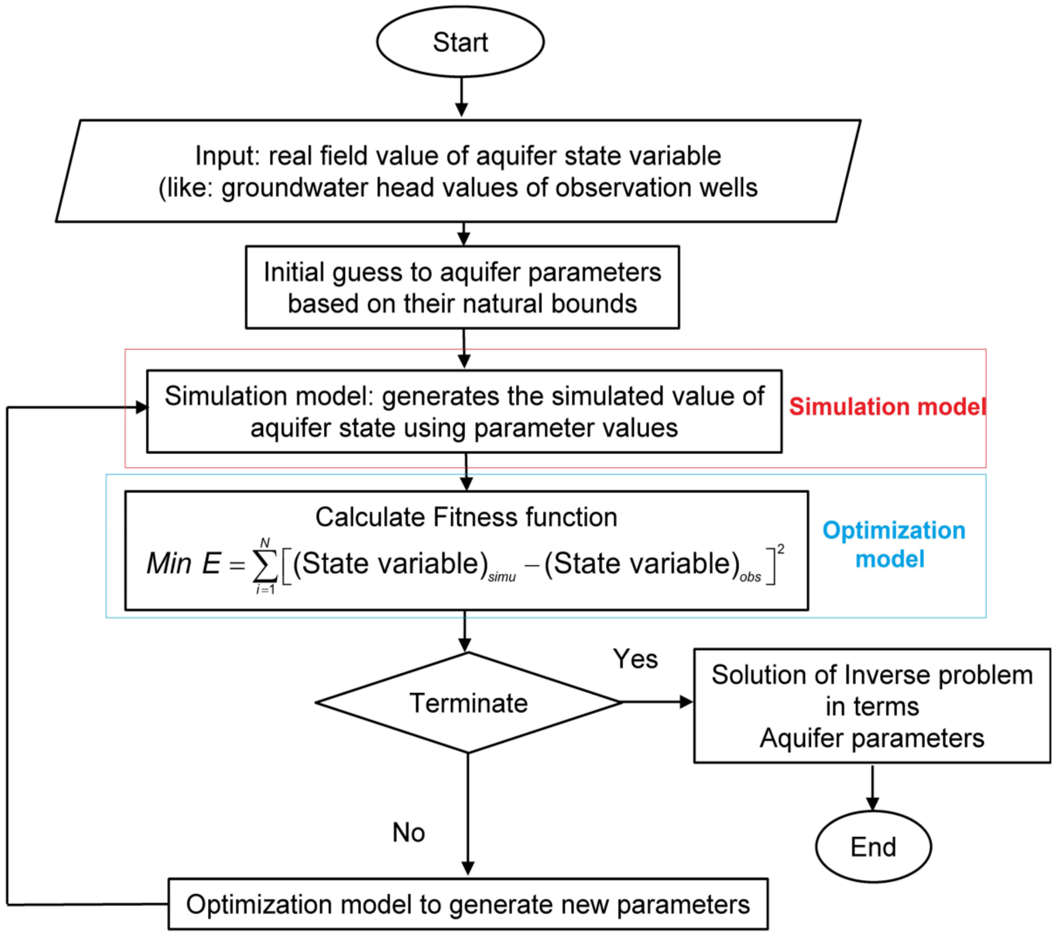


**Fig.3** Flow chart of indirect method.




Yoon and Shoemaker (2001) presented a maiden work on inverse model based on a real-coded genetic algorithm (RGA) for bioremediation problem. RGA results were compared with binary coded GA (BIGA) and were found highly efficient with directive recombination with screened replacement operators during application to two synthetic aquifer bioremediation problems.

Giacobbo *et al.*, (2002) investigated the possibility to use genetic algorithm for estimation of parameters of a groundwater contaminant transport model. In this study, a mono- dimensional advection-dispersion model simulated a three- layered mono-dimensional saturated medium. Here the objective function was the sum of squared residual between pseudo-experimental data, obtained with true values of the parameters, for estimation of velocity and dispersivity parameters. Concentration profiles were computed using estimated parameters. The result obtained through investigation indicated that method is capable of parameter estimation with higher accuracy even in the case of assimilated substantial noise in data.

Tsai *et al.*, (2003a) applied the genetic algorithm with grid search method and quasi- Newton algorithm for parameter identification. Voronoi tessellation method is employed as parameterization method and coupled form of flow and transport equations were solved respectively using tools MODFLOW and MT3DMS.

Snehalatha *et al.,* (2006) developed a model for groundwater parameter estimation (transmissivity) using simulated annealing (SA), a global optimization method. Galerkin finite element method was used for simulating groundwater head values in synthetic rectangular confined aquifer, with the assimilation of the source and sink terms for steady and transient conditions.

Ayvaz *et al.,* (2007) proposed a coupled SO model where kernel- based fuzzy c-means (KFCM) technique was used for parameterization, due to its proficiency to cluster non-spherical shaped data points. Genetic algorithm was clubbed with finite difference method to determine the aquifer parameters and zonal pattern in a synthetic domain. As these heuristic based algorithms generate population (parameters) randomly it is free from the prerequisite of initial guess of parameters which is required in gradient-based optimization.

Gaur *et al.,* (2011) clubbed the Analytical element method (AEM) simulator with Particle swarm optimization (PSO) to solve groundwater management problem. The presented model successfully simulated to get a maximum discharge and minimum cost for the Dore river basin, France.

Jha and Datta (2012) applied the SA as optimization model in contaminant source identification problem. For flow and contaminant estimation over entire domain MODFLOW and MT3DMS tools were used. The main objective of this study was to identify the source and its flux quantification. The obtained results were later compared with the genetic algorithm and were found in the favour of SA.

Jha and Datta (2013) presented adaptive simulated annealing (ASA) based SO model for the determining the groundwater contaminant source character. The results were compared with GA. This study showed that the location of monitoring well is also a very critical parameter to get the best solution. Rastogi and Huggi (2013) applied the heuristic methods genetic algorithm (GA) and simulated annealing for structural characterization of the aquifer. GA was used for estimation of hydraulic conductivity and aquifer recharge for a real field problem, while SA used for a synthetic confined aquifer. Composite scale sensitivity and coefficient of variance were also performed as reliability parameters. After incorporation of noise in the field parameter, estimated results were acceptable from a practical point of view.

Chiu (2014) applied DE as an optimizer to solve the parameter- structure- identification problem. DE effectively minimized the objective function and results showed its robustness and efficiency over the conventional GA. The proposed DE later applied to real groundwater system of Pingtung Plain in Taiwan to identify the aquifer properties.

Gurarslan and Karahan (2015) solved the inverse problem for groundwater pollution identification. They used differential evolution as optimization algorithm to quantify the source in two synthetic aquifers. The first aquifer had the real observation data while the second one was corrupted with noisy data. These results were later compared with an already existing solution obtained through artificial neural network (ANN), GA and Harmony search (HS).

Prakash and Datta (2015) applied the SA for sequential monitoring network design and source identification. In this study initially measured flux from the real field was fed to the forward model to get the concentration value over entire domain for next time step (if identified flux is not satisfactory). Then gradient of pollutant concentration was calculated which was used afterward for designing of a monitoring network for next sampling time step. Therefore on the basis of new well location, the flux values obtained through new well will be feed to the optimization model to identify the source of the pollutant.

Ayvaz (2016) presented a novel hybridized approach engaging binary genetic algorithm and generalized reduced gradient method for source identification problem. It was applied to a hypothetical aquifer problem and the results were compared with genetic algorithm with varying parameters.

In conclusion stated that in order to solve the inverse problem using indirect method various, simulation and optimization models were used, which have their own advantages and shortcomings. Further, based on the survey of literature on different simulations and optimization models briefly presented in Table 1 to 3 the following conclusions can be drawn.

Most of the research work in the field of inverse groundwater modelling is dedicated to synthetic rectangular aquifer domain and very largely without considering actual scenario. These efforts are laudable, however, they are far from the real field problems, which often involve surface recharge, pumping-injection wells, heterogeneity, anisotropy, variable river-heads, boundary flow and aquifer outflows.

Most of the inverse problems are solved iteratively using the indirect method, where the forward problem (simulation) is needed to run many times until desired parameters are not achieved. Many of these old simulation models (FEM and FDM) rely on certain kind of mesh-structures to approximate the field derivative over an entire domain which makes them very expensive in terms of computational and time resources. Therefore the methods without mesh or elemental connectivity can be tested as a simulator for inverse problem solution.

In the past two decades, many evolutionary algorithms of optimization have been applied by researchers to solve a various range of problems in inverse groundwater modelling. Therefore, a comparative study between various heuristic-based optimization techniques i.e. genetic algorithm (GA), particle swarm optimization (PSO), differential evolutional (DE) and covariance matrix adaptation strategy (CMAES) and their hybridized versions for the solution of inverse problems like parameter estimation can be tested.

The conclusions based on the above literature review provided the trend of research gap in the area of inverse groundwater model; hence the same has helped to improve the current research work and provide the direction to the upcoming research works.

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